# Finding the "right" level of abstraction for patterns

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Patterns are abstractions of multiple instances to a core invariant structure of the solution. But what level of abstractions to choose? As each abstraction means a loss of information, we need to think about which abstractions are justified to have patterns that are flexible, open and instructive. We will discuss different types of abstractions and point out what has to be considered in this abstractions process. This will help us to build patterns that are more suitable in their practical usage and to generate a pattern languages that are more consistent and complete.

Categories and Subject Descriptors: I.5.2 [Design Methodology]: Pattern analysis; H.3.1 [Content Analysis and Indexing]: Abstracting methods; D.3.3 [Language Constructs and Features]: Patterns

General Terms: Theory, Documentation, Patterns

Additional Key Words and Phrases: Abstraction methods, emergence, education, knowledge sharing

# 1. WHY IS ABSTRACTION IMPORTANT FOR PATTERN DESIGN?

Patterns generalize over multiple cases and capture the essence of similar structures at a mid-level of abstraction (Gabriel 1996). But "mid-level" is a wide range. Rising (2007) points out there might not be one right level of abstraction. The challenge is that patterns can be too vague or contain too many details (Buschmann 2007). Too detailed descriptions are hard to transfer to new situations. Without experience it is impossible to understand which details of a context-problem-solution rule are essential (e.g. necessary and recurrent) and which ones coincidental. Too abstract pattern descriptions are hard to grasp, difficult to understand and less instructive. A pattern that is too abstract might become meaningless because the relevant parts are missing and we literally do no longer know what the abstract description means.

## 1.1 Challenges

Common challenges a pattern author faces are too abstract pattern descriptions where essential and instructive information is missing. Very often we find different solutions mixed up in one pattern because there may be similarities on an abstract level but many differences in the details. As a result, patterns and pattern languages often switch between different levels of abstraction. Such challenges in abstract representations are not only found in pattern descriptions. We have found that scientific theories, particularly in the humanities, often choose the wrong level of abstraction. Researchers sometimes strive for universal laws where the actual contexts imply a stronger differentiation between different forms. The consequences are that the theoretical descriptions are hard to understand, implement and test. Note that patterns are to some extent theories as well (see Kohls & Panke 2009).

In order to avoid typical mistakes in the process of abstraction we think that it is important to understand the different ways and characteristics of abstraction. We will illustrate these concepts for two running examples. Our first example will be a domain of common sense: traffic systems. In these systems there are many established patterns and they should be very well known (hence, we can save the explicit description). The second line of examples draws from the domain of educational environments. One reason for this line of examples is the experience of the authors and a growing body of educational patterns. The other reason is that educational models and theories are very often on a wrong level of abstraction – either too abstract or too detailed.

In the remaining of this introduction section we will first illustrate some abstractions for both lines of examples. The underlying abstraction strategies are discussed in detail in section 2 and the two lines of examples will

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continue to illustrate these principles. Moreover, we will provide some practical recommendations for pattern authors how to avoid typical mistakes for each abstraction strategy. While section 2 is about avoiding "wrong" abstractions, section 3 will reflect on strategies to find appropriate levels of abstraction.

## 1.2 Common sense abstractions

Let us start our common sense example by considering the pattern of cars. Cars are recurrent structures and they can be used in various contexts of transportation and solve a number of problems (time to travel, weight to carry etc.). The concept of a car is already an abstraction:

- It leaves out specific details such as speed, colour, ownership, current position.
- It generalizes over similar structures, e.g. two cars have the same invariant structure (body, motor, wheels, etc.) but variation in the actual manifestation (form of body, motor type, size of wheels).
- It compounds parts into one larger whole (we can think of cars and their meanings without considering the actual structure).
- It implies functionality and emergent properties (we know about the effects and uses of cars).

CAR is a good level of abstraction and we can describe the concept as a design pattern. It has an invariant structure one can build and "use this solution a million times over, without ever doing it the same way twice" (Alexander et al., 1977). The pattern "car" is generative, real forms can be derived from that concept. The more specific patterns "cabriolet", "SUV", "van" still are generative for there are many variations of these refinements of cars. On the other hand we could describe how to construct a specific car model. A specific model of a car, how-ever, is no longer a design pattern. Being fully specified there is no openness, and the pattern cannot be refined to specific contexts. It is only a template and each instance created by that template looks similar except for surface properties such as colour. An example of a too abstract concept would be a "vehicle". It can mean very different things such as bike, car, train or an airplane. All of these objects belong to the category of "vehicle" but have many structural differences. "Vehicle" is not instructive because it does not reveal the actual structure and consequences.

Another failure of too concrete descriptions is to describe each specific structural part rather than describing a larger whole consisting of smaller parts. It makes more sense to describe the structure in terms of a car body, wheels, motor, brakes etc. Each of these sub parts can be described as another pattern. If we describe a car by defining each single bolt of a wheel, the description would be far too complex. On the other hand we need to ensure that a craftsman either knows where to put the bolts or can find a more specific instruction elsewhere. This means that an abstract instruction such as "has four wheels" needs more specific knowledge on a lower level of abstraction as well. We can say that we unfold the details for each part of the car when we tackle the parts (e.g. where to put the bolts for each wheel). But we do not reveal each of the details when we speak about the structure of the car. Likewise we can omit the actual structure of a car when we describe larger contexts, such as city traffic. If we describe the structure of city traffic we can refer to cars without explaining the actual structure of cars. Yet, if we describe the structure of city traffic, the structure of cars (and other vehicles) is implicitly included. Hence, city traffic has a wider scope than cars, and cars have a wider scope than wheels and bolts.

Still we could say "cars are queueing at a traffic light" without describing the structure of cars and traffic lights at this level of abstraction. We could also say "vehicles are queueing at a traffic light". In a larger context, we are more interested in the emerging functional properties of system parts and not so much on the structural form of the parts. We can refer to "vehicles" rather than "cars" because in the context of city traffic we are more interested in the role of cars, trucks, bikes etc. and not so much in the actual form. However, it is important to understand that the term cars implies specific similar structures (abstracted to a compound), whereas the term vehicle does not imply a specific structure but only specific roles and functions (abstracted to emergent proper-ties). Hence, a vehicle can be part of a higher level pattern (as a structural element that leaves the implementation unspecified) but it is not a pattern itself because it does not define any structure. A car, on the other hand, can also be part of a higher level pattern (as a structural element that implies a specific implementation) and it is a pattern itself because it defines a structural element that implies a specific implementation) and it is a pattern itself because it defines a structure.

Whether we differentiate between cars, trucks, and bikes or consider them all as vehicles is a question of internal granularity (i.e. the "resolution" defines to which detail we differentiate on a certain level of abstraction). Whether we differentiate between car model X, sport cars, cars, vehicles and traffic objects or just between car model X, cars, and traffic objects is a question of external granularity (e.g. the "resolution" defines how many levels of abstraction we have).

## 1.3 Abstractions for educational environments

We can observe the same relations for education. For example, in this field we know the concepts "experiential education" or "experiential learning". While it is certainly a very interesting research field on its own, is it also a suitable concept to teach apprentice-teacher how to prepare for their classroom teaching? We believe not be-cause the variation of actual forms that are considered as "experiential learning" is very diverse. In this field we have a very low internal granularity which means that two very different forms fall into the same category. The reason for this common pitfall is that while the forms are very different, the effects are very similar. Two different forms can have the same emergent effects. To consider all these forms as the same makes the field too abstract in order to specify a teaching design. If we want to be instructive and show how to achieve "experiential education" we need to use a finer granularity and describe the different approaches such as visit, excursion, exploration, hands-on training, internship, project, legitimate peripheral participation (Lave and Wenger 1991) just to mention a few of them (cf. Baumgartner 2011, 267).

On a higher level of abstraction with a larger scope, such as educational systems, it makes sense to refer to emergent concepts such as "experiential education" but we need the differentiation on the next lower level. However, we should also avoid too much detail. If we try to explain apprentice-teachers what to do every couple of minutes and how to sequence these episodes then we surely dig too much into the details. Such explicit plans mostly do not work because nobody can foresee the problems we confront in real life as we cannot dismiss the "thrownness" in our Being-in-the-world as Heidegger put it (2008, 174). Unexpected "interruptions" like a failing technical (teaching) device or a (complicated) question by a student are able to overthrow every de-tailed planning. But even worse: To teach apprentice-teacher on such low level of abstraction prevents to get down to the nitty-gritty of lesson planning. Unfortunately such approach is still very common in teacher education where apprentice-teacher are forced to use for their lesson planning (cf. Figure 1). At this concrete level teacher students are asked to plan their interactions as micro-interventions in the range of some minutes and they have to think ahead what kind of medium and social form they will apply to their micro-teaching episodes (Baumgartner 2006).

Instead of teaching superfluous in-depth details of lesson planning we should concentrate our teaching to teacher-novices how to plan an excursion, an exploration, a project etc. These mentioned approaches aim at some kind of immersion into reality to get his/her own life experiences and are in contrast to many other families of teaching patterns like teacher-centered teaching, problem-based learning, learning through case studies etc. Even if all these teaching models belong to a family of teaching patterns we could call "experiential learning" they are quite distinct in their structure and procedures. They are open for much variation. Two projects for instance are never identical, even if they address the same problems and aim at the same goals (Baumgartner and Payr 1997).

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Figure 1: Forms for lesson structuring and planning (Becker et al. 2007; Böhmann and Klaffke 2010, 21)

## 2. WHAT ARE THE MAIN CHARACTERISTICS OF ABSTRACTIONS?

In the previous two examples of traffic and educational systems we have already used terms such as levels of abstraction, internal and external granularity, scope, compounds, and emergent properties. In this section we will explore these concepts and derive implications for pattern authors.

## 2.1 Different levels of reality

Our examples have shown that there are different levels of abstractions in which we conceive reality. Some philosophers like Nicolai Hartmann (1964) and Michael Polanyi (1969; 1974; 2009) claim that this is not only a perception or construction of our mind but that reality itself consists of different levels. This is a general principle of our world we have to come to grip with. These levels are not just layers ordered one on top of the other, but they form an inclusive hierarchy where the "higher" levels include all the "lower" levels.

## First line of illustration – common sense example car: no. 1

A car includes parts such as a motor, body, chassis, wheels, seat, a steering wheel etc. All these parts could be conceived as members of a "lower" level of the reality of a car. Screws, bolts and fan belt are instances of a layer that is even lower. All objects of a (relatively) lower level are included in the higher level; they form together an inclusive hierarchy with different levels. All these items are tacitly included when we are referring to a "car". In contrast: a driver, a street or a brick on the lane are not parts of a car. These objects belong to different domains.

## Second line of illustration - specialist knowledge example teaching: no. 1

Some educational researcher (e.g. Flechsig (1983; 1996) and Baumgartner (2011)) have also conceived the educational domain as consisting of different levels (Cf. figure 2, showing the inclusion principle by using an "onion"-metaphor instead a pyramid-metaphor). The importance of this conception lies in the claim that different levels are subject to different laws. In order to design educational situations we have to investigate the underlying principles and take them into account.



Figure 2: Inclusive hierarchy of different levels of educational design (Baumgartner 2011, 66)

## 2.2 Emergence

Another important insight is the fact that characteristics of "higher" levels cannot be reduced or explained from the laws or principles of their parts from "lower" levels.

## First line of illustration – common sense example car: no. 2

The mode of operation of a car is not explained by the additive combination of its part. It is the specific organization and interaction of all the parts that cause the car to work.

#### Second line of illustration – specialist knowledge example teaching: no. 2

At the level of curricula we see properties emerge that could not be explained solely from the properties of their modules. Moreover, alternative methods at one level (e.g. educational scenario) could lead to similar effects on the next higher level (educational ensemble). Since we want to investigate the new characteristics and their interdependencies between the different levels we need to be cautious whether we treat different approaches on a lower level as the same on the next higher level.

Looking at different levels, it seems to us that the level of Educational Scenarios is the most important one in preparing teaching classes (We provide the reason for this opinion in section 3). Related to each level of abstraction there are specific sets of questions we need to answer in order to design appropriate interactions: What kind of educational interactions (= lower level) do we need to form a suitable educational scenario? What kind of educational scenarios should we use and how should we orchestrate them into the classroom in order to shape the educational ensemble (= higher level) more effectively for the student's learning experience?

In this conception the chosen ("right") level of abstraction to start with turns out as a middle strata be-cause we have to switch out attention between lower and higher level in order to figure out the necessary building blocks (= lower level) and to estimate the emerging consequences (= higher level).

## 2.3 Ways of abstraction

The layered model of reality has shown that on each higher level we can abstractly refer to more specific parts on a lower level. On a high level description of a traffic system we can refer to cars, traffic lights and streets without providing their details. On a high level curriculum description we can refer to modules without defining their specific educational ensembles. However, there are four different ways to achieve this abstraction.

## 2.3.1 Abstraction by isolation

We can consider a number of different phenomena as identical if we focus on specific features. Features of the actual objects are omitted. According to Erdmann (1892) positive abstraction means to isolate the relevant features and focus on them; negative abstraction isolates irrelevant variations between several objects. Abstraction by isolation is the most intuitive abstraction procedure. Features that are omitted in an abstraction will have no correspondence in the resulting model. Features can be omitted if they are not relevant or it is clear how the appropriate option is selected, i.e. how the best alternative is chosen.

Because we omit irrelevant features, objects that are different in reality can be treated as identical in an abstract representation. For example, consider red, blue and yellow balls. If we abstract from their colour we can consider them as identical. On a low level of abstraction, the red, blue and yellow balls would be differentiated. On a higher level of abstraction all balls are the same.

Patterns authors should focus on structural features that are relevant for a specific question or task. A common misuse of this form of abstraction is to omit features that are essential differentiators. For example, if we want to focus on the nutritional facts of an apple we could abstract from its actual taste. However, if we want to prepare a delicious meal we should not abstract from the actual taste. In this case treating an apple and a lemon as equal could be fatal.

## First line of illustration – common sense example car: no. 3

Each car has an abundance of properties: body colour, maximum speed, production date etc. Which properties matter depend on the questions one wants to be answered. If we are interested in the amount of charge a car can transport, we can abstract from its colour and production date. If we are interested in its aesthetical features we must not abstract from the colour but can abstract from its speed and its transportation properties.

## Second line of illustration – specialist knowledge example teaching: no. 3

Each teaching situation is a complex setup with many properties: the feelings and motivations of students and teachers, the teaching location, the group size etc. However, to describe a teaching situation appropriately we are usually not interested in the height and weight of the students, their names or hair colours.

# 2.3.2 Abstraction by generalization

Once we have isolated relevant features, we can simplify them by abstracting to general values. That means features of the actual objects are covered but not precisely. While abstraction by isolation omits irrelevant features, abstraction by generalization omits irrelevant variations. However, the general structure is preserved (Wundt 1907). A range of different values is mapped to one generalized value. For example, the different widths of a street are mapped to one line thickness on a map. In an abstract representation each value could be represented by its average value or its range of values. Whatever the choice is, the abstract representation no longer differentiates between the exact values.

Patterns authors should be cautious to preserve the structural relations between features. We can generalize over the detailed values of elements but elements and their relations should remain distinct and intact. We can simplify over multiple cases to one general structure (an archetypical or model-like structure) and show

a range of potential configurations rather than specific ones. Since the structural relations are preserved, all emergent properties should be preserved.

#### First line of illustration – common sense example car: no. 4

We can abstract from the actual sizes of wheels, the exact lengths and width of the body etc. However, the structural relations between the parts must remain intact. For example, wheels should be placed at certain positions relative to the car body. Wheels are not larger than the body and the actual values are within a range of valid values. Moreover, at a certain length of a body, we would perceive a limousine or a bus rather than a car.

#### Second line of illustration – specialist knowledge example teaching: no. 4

We can abstract from the actual number of participants, the actual duration of learning sessions or the length of an introduction if these values are within reasonable limits. For instance it may be not relevant whether we organize a learning experience for 10 or 13 participants or whether this experience lasts for 45 or 55 minutes as long as the structural relations of activities remain intact (e.g. one teacher makes the introduction in front of a group of learners).

## 2.3.3 Abstraction to complex compounds

We have isolated the relevant features (abstraction by isolation), and simplified them to general values (abstraction by generalization). The next thing we can do is to abstract interrelated parts to a complex compound, i.e. the complex grouping of features to a whole. Instead of considering the parts of a car, we perceive the car as one whole – a complex concept. Emergence means that the complex interplay of elements of a lower level leads to new effects or laws on a higher level. To understand how a car works we do not need to analyze all its components (Von Baeyer, 2004). The idea of complex abstractions has already been discussed by St. Thomas (cf. Bobik 1963).

On the lower level of abstraction we see the single parts. On the higher level of abstraction, we see the different parts as one whole. Effectively, we introduce a new level of reality - a new dimension. If four points are ar-ranged in the right way they can form a square. However, the "squareness" (the property to be a square) is not due to a single point but the result of the emergent interplay of all four points (King & Wertheimer, 2005). While on the level of points, we have four elements that can be differentiated (four different points), a square is a single element of a higher level. Speaking of squares we do not need to specify the single points anymore to understand what a square is.

## First line of illustration – common sense example car: no. 5

The single parts of a car are integrated into one compound concept on a higher level of reality. Instead of considering the different parts such as wheels, body, seats, motor etc. we consider CAR as one con-cept. It is clearly defined which structural parts cars have. Even if we do not refer to the single elements and do not think of all its structural parts, they can be clearly defined. Speaking of a car means that we can unfold its actual structure.

## Second line of illustration – specialist knowledge example teaching: no. 5

Complex learning situations such as field visit, excursion, exploration, hands-on training, show and tell, internship, projects, brainstorming, or homework review all have a complex underlying structure. Yet we can refer to "excursion" without naming and describing the actual structure. This makes it easier to use the concept of "excursion" in higher levels of abstractions. However, one can clearly map "excursions" to an underlying structure.

Patterns should describe the complex compound of a generalized structure. A pattern captures both the underly-ing structure and the emergent effects, including the positive and negative consequences. The parts of a pattern must form one coherent whole. It is also important that the structure is preserved. The pattern should be instructive, i.e. contain enough information to actually generate the underlying structure of the complex compound that is perceived as one whole - one gestalt.

## 2.3.4 Abstraction to emergent properties

Complex compounds abstract from the actual structure and focus on their emergent gestalt. We can still map the complex compound to its actual parts. Now we can also go one step further and abstract from the actual parts and only consider the emergent properties. On a higher level of abstraction this provides us a certain degree of freedom because we are not bound to one specific implementation (for example, if we refer to "vehicle" we are not bound to the structure of cars). On the lower level of abstraction, however, we have a degree of uncertainty because we can no longer say which actual forms serves on the higher level (e.g. when we refer to "vehicle" we might find a car or a bike which have very different structures).

In object oriented programming it is quite common to have interfaces that abstract from the actual implementation. The interfaces describe the behaviour on a macro-level and the actual structure of the implementation (micro-level) is encapsulated and invisible for the user. Abstraction to emergent qualities is discussed in the con-text of information and complexity theory (for example: Gleick 2011; Mitchell 2009; Holland 2000). The structure of the lower level is no longer preserved on the higher level. An abstraction to emergent qualities considers the actual micro-structure on a lower level as a "black box" and abstracts to the behaviour in a macro-structure of a higher level. We no longer consider the details of the actual parts but the emerging qualities. On the lower level of reality objects may be very different whereas on the higher level different objects may have the same effect.

#### First line of illustration – common sense example car: no. 6

Different vehicles can transport us to a destination. If we describe a complex traffic system we can refer to any type of vehicles instead introducing specific cases for all the different vehicles. For example, when we consider the case of queueing in front of a traffic light, it makes more sense to refer to all vehicles instead of considering all the different cases.

## Second line of illustration – specialist knowledge example teaching: no. 6

An educational scenario could promote at one point of time the exploration of concepts. However, the structure of an exploration activity can be very different. On the macro-level we would expect the same effects of the exploration activity (reflect, find new insights) but on the micro-level we can find many different forms of exploration (web research, experiments, group discussion). To implement any of these specific forms of exploration, a teacher needs to know different sequences of steps. Hence, refer-ring to "exploration" provides freedom of choice but it also requires prior knowledge about some forms of "exploration".

If we take two complex compounds, they can be very different in their structure but have similar emergent qualities. Pattern authors should be aware that two different solutions can solve the same problem. While these two solutions share similar emergent properties (effectively solving the problem) they do have different structures and may have different consequences. If you put multiple different solutions into one pattern description, it becomes harder to read. If you describe only the similar emergent properties then there is no mapping to the actual underlying structures. But design patterns should help the reader to generate actual structures. Moreover, if you focus on the emergent similarities, you can easily forget that there are other emerging properties (side effects or consequences) that can be different for each of the solutions.

Actually if you take two different compounds they are likely to have many different emergent qualities as well. For example, cars and bikes are both means of transport but their use has many different consequences. Like-wise, two educational methods of exploration may have different side effects and yet serve the same purpose. Obviously we start the process of isolating relevant features for another round. This time, however, the abstraction is done for the emergent properties, i.e. on a new level of reality. On a lower level of abstraction the structure of cars and bikes is very different. On a higher level of abstraction we can find emerging qualities that are similar. On this next higher level, we can again abstract by isolation, generalization and by finding complex com-pounds. The new complex compounds will have once again emergent properties, and the process can start again.

## 2.3.5 Granularity and scope

In choosing the appropriate level of reality for pattern construction we have to account for two other features in the inclusive hierarchy of abstractions. One is the distance between different objects at the same level of abstraction and the other one is the distance of the different levels themselves. We will call the first one internal granularity and the second one external granularity.

As we have seen there is a recursive process, involving different types of abstractions, that leads to an inclusive hierarchy of abstract entities. The result depends on our decisions what abstractions we perform. Depending on which features are omitted and which ranges of values are considered to be similar, we will have a different number of abstraction levels (external granularity) and a different number of entities that are treated as similar (internal granularity).

Internal granularity measures to what extent a certain level of abstraction is populated with objects or processes. The more items we account for the abstraction level in question the higher is the chance that two items are quite similar and less far away in our internal graded cognitive representation.

External granularity on the other hand is a measure how far away two levels of abstractions are situated. Granularity can be compared with the digital composition of a photo: internal granularity looks at the chosen level of pixel resolution, external granularity focuses at the texture - the overall structure of the picture.

## First line of illustration – common sense example car: no. 7

*External Granularity:* We have a coarse external granularity when we construct a hierarchy with just two levels of "vehicle  $\leftarrow \rightarrow$  British sport car" as compared to a finer granularity that consists of the abstraction levels "vehicle  $\leftarrow \rightarrow$  motor vehicle  $\leftarrow \rightarrow$  personal motor vehicle  $\leftarrow \rightarrow$  car  $\leftarrow \rightarrow$  sport car  $\leftarrow \rightarrow$  British sport car". In the second case the "space" between vehicle and British sport car is filled up with other levels of abstraction and reduces the distance or gap between different levels. It depends on the problems we want to solve and our target group we want to address what would be the more appropriate break down of reality into abstractions levels.

Internal Granularity: If we consider one of our specified abstraction levels (e.g. "car") the difference in internal granularity would be exemplified by "cars  $\leftarrow \rightarrow$  trucks  $\leftarrow \rightarrow$  bikes" versus "car model 1  $\leftarrow \rightarrow$  car model 2  $\leftarrow \rightarrow$  car model n". On a lower level we could distinguish between brands or even years of production.

## Second line of illustration – specialist knowledge example teaching: no. 7

*External granularity:* We have already argued for a greater external granularity for training purposes of apprentice-teachers. Instead of combining the tiers of "education scenario  $\leftarrow \rightarrow$  education ensemble  $\leftarrow \rightarrow$  module" into one category that may be called "didactical design", "teaching" or "instruction", we suggest cutting reality into more tractable (smaller) layers and breaking up the very general category of "teaching".

*Internal granularity:* The more entities that are ascribed to one level the finer graduated are the differences. Again it depends on our educational goal and the target group we want to address.

One could argue that novice-teachers should not be confronted with too many details, as they could be intimidated purely from the sheer amount of different options. So we should constrain our instruction to novice-teacher to the prototype or some very high ranked "good" examples. On the other hand one could counter that the complexities of reality should not be concealed completely because it would make it more difficult to find answers to practical challenges. Whatever argument is valid it is clear enough that to educate expert teachers one certainly would need a more refined abstraction layer with more objects or processes e.g. a bigger family with more members.

#	Layers of Educational Actions	Levels of Educational Abstractions							
		Descrip- tions	Met	hods	Principlos	Dimen- sions	Cate-		
			Patterns	Models	Fincipies		gories		
		1	2a	2b	3	4	5		
н	Internat. Systems								
G	Nat. Systems								
F	Institutions								
E	Curricula								
D	Modules								
с	Ensembles								
в	Scenarios								
Α	Interactions								

Figure 3: Educational Taxonomy (Baumgartner 2011)

One of the authors (Peter) has presented in his educational framework (Baumgartner 2011) several lev-el of abstractions for every different level of education action. This demonstrates that there could al-ways be different level of abstractions to describe the same educational situation.

We have already described the levels of the y-Axis as an inclusive hierarchy (cf. figure 2), so we can concentrate on the x-Axis of figure 3: The hierarchy of abstractions levels consists of five layers: (educational) Categories  $\leftarrow \rightarrow$  (educational) Dimensions  $\leftarrow \rightarrow$  (educational) Principles  $\leftarrow \rightarrow$  (educational) Methods  $\leftarrow \rightarrow$  (informal educational practice) Descriptions. Each of these different abstractions levels has a different scope and is addressed to different design problems and targets groups. Educational categories form the most general abstraction level and address the design of educational theories. On the other pole we have informal educational descriptions lacking abstractions at all but are suitable for personal day-to-day communication. One essential line of argument in this hierarchy of abstractions is the claim that the formal format of education patterns are better suited to transfer practical know-how than the more abstract and stale format of teaching models.

Whether two entities are considered to be similar or not has impact on our options for building compounds (Kohls, 2011). Let's say we have the entities A, B, C1 and C2. If we consider C1 and C2 as similar and abstract them to C (coarse internal granularity), then we can form one compound ABC. Hence, we can limit our levels of abstraction to a higher level of ABC and a lower level of A, B, C1, C2. The alternative would be to have another level in between that consists of ABC1 and ABC2 (three levels of abstractions mean a finer external granularity).

Entities that are complex compounds of smaller entities have a wider scope. For example, the compound ABC has a wider scope than each of its sub parts A, B, C1 or C2.

Scope is another important technical term in our argumentation. Scope indicates the share of influence to change reality accounted for by a certain abstraction level. Scope determines the relative position in the inclusive hierarchy of abstractions. Scope depends on the focus of our interest and the target group we plan to ad-dress.

*Scope* refers to the realm of reality and the circumference covered by the level of abstraction in question. De-signing patterns of lower levels of abstractions are linked to seemingly smaller changes if we just look at the objects or processes themselves. But as a new design of these smaller items could also change their interaction with other objects or processes in the world they could have (through emergence) profound impact on higher levels of reality. For instance the appearance of small gadgets like the smartphone has changed significantly our communication habits and life style.

## First line of illustration - common sense example car: no. 8

If we design patterns for a traffic solution each pattern would have a much greater impact (=broader scope) than we would have if we would focus just on the design of passenger cars. For the traffic solution we would also need to incorporate the population, housing, structure of streets, parking places, traffic lights, public transport etc. In the design of passenger cars all these items are of minor relevance.

## Second line of illustration - specialist knowledge example teaching: no. 8

The design of educational policies by the government has a much broader range of impact than the design of micro-didactical interventions by a teacher. If we want to cover such a broad field, we need to cope with the complexity on a higher level of abstraction. In the scope of educational policy it is appropriate to abstract to experiential learning and only consider the emergent impacts in the context of the whole policy. However, if we want to design courses that include experiential learning we are changing our scope to that part of the larger whole. Therefore, we need to change to a granularity level that makes the differences between visit, excursion, exploration, hands-on training, internship, project etc. visible.

Pattern authors should be coherent in their levels of abstraction. The most important thing about granularity is not the absolute size of the distance of their items but their consistency throughout the different abstraction levels. If there are big differences in the number of objects or processes populating the different levels we may ask if we have broken up the levels of reality appropriately. The division of a larger whole into its parts has not only implications to the gap between the different levels of abstraction (external granularity) but has also con-sequences for internal granularity. If the overall structure is very coarse it does not make sense to go into the very details of just one of those levels.

# 3. HOW TO CHOOSE THE MOST APPROPRIATE LEVEL OF ABSTRACTION?

If there are different levels of reality, which we frame cognitively as abstractions, one question arises: How can we detect the most appropriate level we should focus on in order to solve a certain problem? Some of these levels are more adequate for our design tasks than others. There are also some levels that are more adequate for teaching purposes as they are easier to grasp. Is there only one suitable level for a special problem we want to address? Certainly the subject area where our problems are situated in is important for our decision as well as the target group to which our teaching patterns are directed. But this is trivial and is not enough to specify the most suitable level of reality we should focus on abstractions.

We believe that the main criteria to choose a proper level of abstraction are embedded in real life encounters. Both lines of exemplifications show that we need to confront a real life object or process. Neither "vehicle" nor "experiential learning" qualifies in this respect. But to be embedded in real life is not enough. The object or pro-cess has to be a self-contained whole, which is independent and has – so to speak – a life of its own. Neither "auto body" nor "micro-intervention" qualifies to this restriction.

# 3.1 Abstraction and models

An abstraction is a mapping from the actual world to a model (Holland, 2000, p. 30). Abstraction by isolation and abstraction by generalization are intuitively understood because they map reality to models in a static

way. However, to model the interplay of elements and their emergent properties we need to think about dynamic models:

- Concrete models: These are real, physical objects or processes intended to map or represent some generalized phenomenon. A model can even be a structure that actually exists in the real world, i.e. a mod-el citizen, a model student or a model school. In such cases the model is just an instance of the class it models (Goodman 1976).
- Mental models: A model can be a mental construct that represents real, hypothetical, or imaginary situations (Johnson-Laird 1983). In this group we will find the important group of mathematical models.
- Computational Models: A model can also be something that we artificially create, e.g. a simulation or a model of molecules to better grasp their structures. In this dynamic case we try to map the behaviour of a system and have therefore also to account for the impacts of interactions (Weisberg 2013).

Not only the computational model but also the concrete and mental models can be used to map system behaviour. In that case the isolation and generalization of features alone is not adequate because these types of abstractions cannot account for new features, which may emerge through the interactions of their elements. A dynamic model preserves the quality that emerges from the complex interaction of the elements of a sub-structure. This mutual dependency is not explicit but implicit in the order of each instance (Bohm 1981). The structural quality of the form emerges from the interplay of its part and not from the single positions of the parts. A global behaviour that outlasts any of its components is a defining characteristic of complex systems (Johnson 2002, 82). The order is not defined by a statistical distribution of each part but the result of self-organizing parts. Elements influence each other and their configurations feedback to the configuration of other elements.

## 3.2 Basic categories

Our assumption can be backed up with empirical research in cognitive psychology. Eleanor Rosch and her colleagues have shown that there are basic levels in cognitive categories (Rosch and Lloyd 1978). When people are asked, "What are you sitting on?" they prefer to say "chair" rather than a concept of a lower abstraction level like "kitchen chair" or of a higher level such as "furniture".

Basic categories – according to the theory Rosch & colleagues had worked out – are characterized by relatively homogeneous sensory-motor affordances. A chair can be associated with bending of one's knees, but the lower abstraction levels like kitchen chair do not add anything important to the basic level of our (body) involvement with reality. And the higher abstraction level is too abstract to imagine (e.g. visualize) some concrete interaction with a real object.

## First line of illustration – common sense example car: no. 9

We do have interactions with vehicles but the concept is too broad to imagine specific kinds of interactions. These necessary interactions for riding a bike or drive a car are so different that we cannot subordinate all of them in an effective cognitive representation. There is no visual presentation, no gestalt of a self-contained whole for "vehicle".

On the other hand: Driving a car needs certain cognitive and motoric operations. They are all pretty well covered by the general concept of a car. The differences between different passenger cars of different brands do not add any significant feature of movement or interaction. Even the different cognitive and motoric challenges of a passenger car and a truck are of minor importance for our internal presentation of a car.

## Second line of illustration – specialist knowledge example teaching: no. 9

What is the "basic level" of teaching? Looking at Figure 2 we certainly can dismiss the two highest levels of abstraction: Politics and institution. Writing curricula is maybe also a certain candidate for exclusion as specialists mainly do this work or – even when teacher design their own curriculum – they do this not regularly on a daily basis.

If we investigate the bottom of the hierarchy we mentioned already that micro interventions (level of educational interactions) is not suitable for educational design. So there remain the levels of education-al scenarios, educational ensemble and modules. Educational scenarios are in educational textbooks listed under the concept of teaching methods. In our taxonomy we understand as methods for example group work, fish bowl, brainstorming but we also include methods to implement a certain variety of experiential learning like visit, excursion, exploration, hands-on training, internship, project and legitimate peripheral participation. But we do not include experiential learning or teacher-centered teaching, problem-based learning, and learning through case studies. These concepts do not have a direct and concrete cognitive and corporeal involvement of teachers and students. They are not basis categories of teaching.

Similar considerations apply for the levels of educational ensembles and modules. In educational ensembles teacher put together their different teaching methods they have used to reach a predefined teaching goal in a certain subject matter area. And modules are the European parlance for the level of assessment, where student performance has to be graded.

So it turned out that there are three levels of abstractions where teacher and students are involved cognitively and bodily: scenarios, ensembles and modules. All three of them could work as basic teaching level. So why not group them together? We claim that this detailed break up of reality into three abstraction levels is necessary as these levels are liable to different laws and educational principles. It easier and more convenient educate apprentice-teacher when we address each of these categories separately. It could be argued that one big problem of the educational sciences lies in the fact that these different layers of abstractions are not observed accordingly.

Taken our example of lesson planning it is obvious that educational ensembles or even modules are too big, because they do not fit in the still (at least in our countries Austria and Germany) predominant 40-50 minutes time frame of teaching units. This is the reason why there are so many German books on teaching methods und lesson planning. At the same time we are lacking books on content blocks (Educational Ensembles in our parlance) and assessments of learning (Modules in EU parlance). In the follow up of the PISA-studies – where the German speaking countries have bad rankings – one line of dis-cussion criticizes these small teaching units, as they require a relatively low level of planning. It is argued that German-speaking countries need to revise the structure of their educational framework similar as the PISA high ranked countries like Finland had demonstrated (Sahlberg 2011).

## 3.3 Graded categories, prototypes and family resemblance

In other experiments it turned out that people do have privileged concepts in their mind, which they connect strongly to a certain level of abstraction. For instance if probands were presented objects like a chair or telephone and had to decide how good these items could perform as examples for furniture, they came up with a ranking where chair, sofa, couch and table are the top ranked items and sewing machine, stove, refrigerator and telephone are those item least connected to the abstraction "furniture" (Rosch 1975).

These and similar results led to prototype theory: Instead of clear-cut categories in the Aristotelian sense where all items of a certain category share some of their properties Rosch and others claimed that there are graded categories where some objects are more central to this category than others.

The cognitive linguist George Lakoff (1987) added the idea of family resemblance as another supporting evidence of prototype theory. The concept of family resemblance was developed by Ludwig Wittgenstein (1961) and claims that members of a specified category are not simply related by sharing similar features but linked by a chain of intermediate members with whom they do share some features. So it could happen that one family member of a certain category, situated far away from the prototype, does not even share one single property with another member of this category, which also can be conceptualized with a long distance to the central ex-ample (=prototype).

The primary common sense example given in this respect is "bird". In the Aristotelian understanding a bird may be defined with common properties like feathers, a beak and the ability to fly. In contrast, prototype theory would consider a category like bird as consisting of different elements which have unequal status – e.g. a robin is more prototypical of a bird than, say a penguin.

## First line of illustration - common sense example car: no. 10

There are many different cars but when we try to visualize one the chance that we imagine a normal passenger car is the highest. SUVs, pickups or trucks are not typical examples; they do not form the prototype of our concept of "car". Cleary this could change with future development and usage behaviour and there are also some cultural differences to keep in mind.

## Second line of illustration – specialist knowledge example teaching: no. 10

It is much more difficult to visualize a typical method for experiential learning. One reason for this difficulty is that teaching methods are no objects that one can touch, grasp or shape visually. Another rea-son is that in our daily teaching practice – at least in the German speaking countries – experiential learning approaches are still not very common. Studies show that about two-thirds of teaching interaction is still done in the teacher-centered presentation mode (Seifried and Klüber 2006, 8) We still visualize a typical learning situation with the teacher in the front of the classroom talking only supported from time to time by some media (blackboard, computer and beamer, interactive whiteboard etc.).

If we would plan an experiment and teacher present different methods of experiential learning such as visit, excursion, exploration, hands-on training, internship, project, legitimate peripheral participation we would perhaps get a sound ranking. We assume that the methods of "hands-on training" or "internship" rank higher (e.g. are more prototypical for experiential learning approaches) than visit or excursion. But it could also be that "project" would get the highest rank as this method is well known and can be integrated into "standard" teaching relatively easy. We are not sure as we are lacking empirical data.

To look for features, which are common in multiple instances, there is a tendency to seek for a common denominator. But we have learned from prototype theory and from the concept of family resemblance that this kind of procedure does not cover reality. As a result rich concepts and their many variations are reduced to a com-mon denominator.

The members of a category are not simply related by sharing identical or similar features but linked by a chain of intermediate members with whom they do share (some) features. Two instances (family members) are there-fore related to each other, but not by abstract commonalities, rather by being unfolded from the same primal phenomenon (so to speak from the same "ancestor" if we stick with the metaphor of "family resemblance").

# 4. GRASPING THE WHOLENESS, THE GESTALT

Taken this argumentation into account we have to distinguish between two different kinds of "right" levels. One type is just motivated by a certain practical problem under a given set of inclusive hierarchical strata. Here we ask: "What is the appropriate abstraction stratum to solve the problem in question?" The other type of "right-ness" is stimulated by a consideration on the system level: "Do we have the appropriate structure and frame-work to solve the posed problem?" This distinction could be boiled down to the famous dictum by Peter Drucker (1963): "doing the right things and doing things right". "Doing the right things" is committed to an overall per-spective and is therefore more important than "doing things right". But not always are we in the position to de-sign the architecture of the whole system and then we have to limit our efforts for the solution of a problem under preset conditions.

The important consequence for our argumentation for choosing the "right" level under a given framework is the following conclusion: The "right" level of abstraction is one that is linked with basic human categories and is rich enough to consist of a manageable amount of family members which assures variety not only in details but also in important and characteristic traits. Our suggestion is that at least 20-25 family members could be

a good starting point and Dunbar's number – which lies "between 100 and 230, with a commonly used value of 150" ("Dunbar's Number" 2013) an appropriate endpoint. Dunbar's number is a suggested cognitive limit to the number of people with whom one can maintain stable social relationships. This seems also a good approximation to the metaphor of family resemblance: Is the family bigger than Dunbar' number she is not manageable any-more, the family ties weaken and the community stops to function as a family. More than 150 family members in the chosen level of abstraction might already go too much into details for writing useful and practical patterns for a certain domain level.

When we write patterns we always should consider the impacts for the higher and lower hierarchical level of abstractions: Keeping in mind this triad of hierarchical levels means that our focus is always on the (relatively seen) medium levels. We will always see members of that category, which show both similarity and variation. The similarity is due to the fact that members of that category have taken the same paths in their development e.g. in their history of unfolding. The variation in contrast is due to the fact that the process of unfolding on that level has not ended and therefore the members still unfold in different ways.

The term "unfold" signifies that the possibilities of the future development are already there but has still not evolved. It depends of different effects if these hidden properties have a chance to develop and it what direction they may evolve or mature. Writing patterns mean to catch the wholeness already intrinsically present. To grasp this essence, the whole form we need to look into the past (lower level of abstraction) and the prospective future (higher level of abstraction). Only then we will have a chance to start to grasp the whole form – the gestalt. The whole is already present as a nucleus in each concrete instance. More and more wholeness is gained in the process of unfolding.

More concrete representations capture more of the wholeness ("Gestalthaftigkeit"). A very concrete representation shows a fully unfolded instance that is no longer open to differentiations in the potential designs ("Gestaltbarkeit" is missing) (Kohls 2009). It is therefore, that patterns are abstractions on a medium-level e.g. have to be written for a (relatively seen) medium level in the hierarchy of abstractions we are interested in. They are concrete enough to let us see the whole (the "gestalt") while at the same time they are not finite instantiations. While there are abstract representations used to communicate and capture patterns, the actual pattern includes all the forms possible. A pattern constrains the forms but it does not fully define them. It is a form category and not just an abstract form.

## ACKNOWLEDGEMENTS

Many thanks to our shepherd Antonio Maña. We gave him a hard time as this paper is not an easy read (even the authors still struggle to understand some of the parts...). It is a journey, and our views and insights are still changing...therefore the patience and guidance of Antonio is much appreciated.

#### REFERENCES

Alexander, C., Ishikawa, S., Silverstein, M., Jacobson, M., Fiksdahl-King, I., & Angel, S. 1977. A pattern language. New York, USA: Oxford University Press.

Baumgartner, Péter. 2006. "Unterrichtsmethoden Als Handlungsmuster-Vorarbeiten Zu Einer Didaktischen Taxonomie Für ELearning." In DeLFI, 4:51–62. http://peter-baumgartner.at/schriften/article-de/handlungsmuster-taxonomiepdf.pdf.

Baumgartner, Peter. 2011. Taxonomie von Unterrichtsmethoden: Ein Plädoyer Für Didaktische Vielfalt. Münster Westf: Waxmann.

Baumgartner, Peter, and Sabine Payr. 1997. "Methods and Practice of Software Evaluation. The Case of the European Academic Software Award." In Proceedings of ED-MEDIA 97–World Conference on Edu-cational Multimedia and Hypermedia. http://medida.bildungstechnologie.org/mdd\_2005/mdd\_2001/easa-evaluation.pdf.

Baeyer, Hans Christian von. 2004. Information: The new language of science. London: Phoenix.

Becker, Gerold, Andreas Feindt, Hilbert Meyer, Martin Rothland, Lutz Stäudel, and Ewald Terhart, ed. 2007. Guter Unterricht. Maßstäbe Und Merkmale - Wege Und Werkzeuge. Vol. Friedrich Jahresheft XXV. Seelze: Friedrich Verlag.

Bobik, Joseph. 1963. "Pattern Recognition Mechanisms and St. Thomas' Theory of Abstraction." phlou Revue Philosophique de Louvain 61 (69): 24–43.

Bohm, David. 1981. Wholeness and the Implicate Order. London; Boston: Routledge & Kegan Paul.

Böhmann, Marc, and Thomas Klaffke. 2010. Die Neuen Kommen! Gut Starten in Schule Und Kollegium. Supplement Zum Friedrich Jahresheft. In Friedrich Jahresheft. Seelze: Friedrich Verlag.

Buschmann, Frank. 2007. Pattern-oriented Software Architecture, Vol. 5,. Chichester, England; Hoboken, N.J.: Wiley.

Coplien, James O. 1996. Software Patterns. New York; London: SIGS.

Drucker, Peter F. 1963. Managing for Business Effectiveness. Boston, Ma.: Harvard Business Review Reprint Service.

"Dunbar's Number." 2013. Wikipedia, the Free Encyclopedia.

https://en.wikipedia.org/w/index.php?title=Dunbar%27s\_number&oldid=566409456.

Erdmann, Benno. 1892. Logische Elementarlehre. Max Niemeyer.

Flechsig, Karl-Heinz. 1983. Der Göttinger Katalog Didaktischer Modelle : Theoretische Und Methodologische Grundlagen. Göttingen. Nörten-Hardenberg: Zentrum f. didakt. Studien.

Flechsig, Karl-Heinz. 1996. Kleines Handbuch Didaktischer Modelle. Eichenzell: Neuland Verl. für Lebendiges Lernen.

Gabriel, Richard P. 1996. Patterns of Software: Tales from the Software Community. Oxford University Press Inc, USA.

Gleick, James. 2011. The Information: a History, a Theory, a Flood. New York: Pantheon Books.

Goodman, Nelson. 1976. Languages of Art: An Approach to a Theory of Symbols. Indianapolis: Hackett.

Hartmann, Nicolai. 1964. Der Aufbau Der Realen Welt. Grundriß Der Allgemeinen Kategorienlehre. 3rd ed. Gruyter.

Heidegger, Martin. 2008. Being and Time.

Hofstadter, Douglas R. 1979. Gödel, Escher, Bach: An Eternal Golden Braid. New York: Basic Books.

Holland, John. 2000. Emergence : from Chaos to Order. Oxford: Oxford University Press.

Johnson, Steven. 2002. Emergence: the Connected Lives of Ants, Brains, Cities, and Software. 1st Touchstone ed. New York: Touchstone.

Johnson-Laird, Philip Nicholas. 1983. Mental Models: Towards a Cognitive Science of Language, Inference, and Consciousness. Harvard University Press.

King, D. B., & Wertheimer, M. 2005. Max Wertheimer & Gestalt theory. New Brunswick: Transaction

Publishers.

Kohls, Christian. 2009. "E-Learning Patterns - Nutzen und Hürden des Entwurfsmuster-Ansatzes." In E-Learning 2009: Lernen im digitalen Zeitalter, edited by Nicolas Apostolopoulos, Hoffmann Harriet, Veronika Masmann, and Andreas Schwill, 61–72. Münster: Waxmann Verlag.

Kohls, Christian. 2011. The Structure of Patterns - Part II: Qualities. PLoP 2011 - 18th Pattern Languages of

Programs conference. Writers' Workshop version. Portland, Oregon.

Kohls, Christian, & Panke, Stefanie. 2009. Is that true? Thoughts on the epistemology of patterns. Proceedings of the 16th Conference on Pattern Languages of Programs. Chicago: ACM.

Lakoff, George. 1987. Women, Fire, and Dangerous Things: What Categories Reveal About the Mind. Chicago: University of Chicago Press.

Lave, Jean, and Etienne Wenger. 1991. Situated Learning: Legitimate Peripheral Participation. Cambridge University Press.

Mitchell, Melanie. 2009. Complexity: a Guided Tour. Oxford [England]; New York: Oxford University Press.

Polanyi, Michael. 1969. Knowing and Being: Essays by Michael Polanyi. Univ of Chicago Pr.

Polanyi, Michael. 1974. Personal Knowledge: Towards a Post-critical Philosophy. Corr. Ed. University of Chicago Pr.

Polanyi, Michael .2009. The Tacit Dimension. Reissue. University of Chicago Press.

Rising, Linda. 2007. "Understanding the Power of Abstraction in Patterns." IEEE Software 24 (4) (August): 46–51.

Rosch, Eleanor. 1975. "Cognitive Representations of Semantic Categories." Journal of Experimental Psycholo-gy: General 104 (3): 192–233. doi:10.1037/0096-3445.104.3.192.

Rosch, Eleanor, and Barbara L. Lloyd. 1978. Cognition and Categorization. John Wiley & Sons Inc.

Sahlberg, Pasi. 2011. Finnish Lessons: What Can the World Learn from Educational Change in Finland? New York: Teachers College Press.

Seifried, Jürgen, and Christina Klüber. 2006. Unterrichtserleben in Schüler- Und Lehrerzentrierten Unterrichtsphasen. Konstanz. Bibliothek der Universität Konstanz,.

Weisberg, Michael. 2013. Simulation and Similarity: Using Models to Understand the World. Oxford Univ Pr.

Wittgenstein, Ludwig. 1961. Philosophical investigations. New York: Macmillan.

Wundt, Wilhelm. 1907. Logik der Exakten Wissenschaften. Stuttgart: F. Enke.